Sec. 13.4:

Motion in Space: Velocity and Acceleration

What We Will Go Over In Section 13.4

1. Motion in Space: Velocity and Acceleration

1. Motion in Space: Velocity and Acceleration

- The graph of a vector-valued function is a space curve.
- The graph of a vector-valued function can be thought of as the path of a fly as it flies around in 3-space.
- In section 13.4, we will use t as the input variable for vector valued functions and think of it as standing for time. So…

  The input t is a time

  and…

  the output is the position (or location) of the fly at that time

1. Motion in Space: Velocity and Acceleration

<u>Recall</u>:

In Calc. 1 when objects were only allowed to move back and forth in a straight line…

- The derivative of an objects position function $s(t)$ is its velocity function $v(t) = s'(t)$
- The derivative of an objects velocity function $v(t)$ is its acceleration function $a(t) = v'(t) = s''(t)$

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The idea in Calc. 3 is the same, except now the object is not confined to a line. It can travel anywhere in space.

- The derivative of an objects position $\vec{r}(t)$ function is its velocity function $\vec{v}(t) = \vec{r}'(t)$
- The derivative of an objects velocity function $\vec{v}(t)$ is its acceleration function $\vec{a}(t) = \vec{v}'(t) = \vec{r}''(t)$

<u>Note</u>: The speed of the object is the magnitude of its velocity

$$speed(t) = |\vec{v}'(t)|$$

1. Motion in Space: Velocity and Acceleration

Ex 1: The position vector of an object moving in a plane is given by $\vec{r}(t) = t^3\boldsymbol{i} + t^2\boldsymbol{j}$. Find its velocity, speed, and acceleration when $t = 1$ and illustrate geometrically.

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Ex 2: Find the velocity, acceleration, and speed of a particle with position vector $\vec{r}(t) = < t^2 , e^t, te^t >$.

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Ex 3: A moving particle starts at an initial position $\vec{r}(0) = < 1, 0, 0 >$ with an initial velocity $\vec{v}(0) = \boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$. Its acceleration is $\vec{a}(t) = 4t\boldsymbol{i} + 6t\boldsymbol{j} + \boldsymbol{k}$. Find its velocity and position at time t.

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Ex 4: An object with mass m that moves in a circular path with constant angular speed ω has position vector $\vec{r}(t) = a\cos\omega t\ \boldsymbol{i} + a\sin\omega t\ \boldsymbol{j}$. Find the force acting on the object and show that it is directed towards the origin.